Department of Mathematics

MTL 106 (Introduction to Probability Theory and Stochastic Processes)

Minor 1 (I Semester 2016 - 2017)

Time allowed: 1 hour　　　　Max. Marks: 25

1. (a) Write axiomatic definition of probability. (3 marks)

   (b) Let $\Omega = \mathcal{N} = \{0, 1, \ldots\}$. Let $\mathcal{F}$ be the largest σ-field on Ω. Define a probability measure on $(\Omega, \mathcal{F})$ by $P(\{n\}) = k2^{-n}$ where k is a constant.
   (i) Find k. (ii) What is the probability of the event $\{n \in \Omega : n \text{ is even }\}$?. (1 + 1 marks)

2. A continuous random variable X has pdf $f(x) = \begin{cases} \beta x & 0 \leq x < 1 \\ \beta & 1 \leq x < 2 \\ -\beta x + 3\beta & 2 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$.

   (a) Determine the value of β.

   (b) Find x such that $P(X \leq x) = 0.5$. (2 + 3 marks)

3. State **True** or **False** with valid reasons for the following statements. Without **valid reasons**, marks will NOT be given.

   (a) Let $\Omega = \{a, b, c\}$. If $F_1 = \{\emptyset, \{a\}, \{b, c\}, \Omega\}$ and $F_2 = \{\emptyset, \{a, b\}, \{c\}, \Omega\}$ are two σ-fields on Ω, then $F_1 \cap F_2$ is a σ-field on Ω.

   (b) Suppose the overall percentage of A grade in MTL 106 examination is 80. If 6 students appear in the examination, the probability that no student will get A grade in the examination is $(0.2)^6$.

   (c) Define the $(100p)$th percentile of a random variable X is the smallest value of x such that $F(x) = P(X \leq x) \geq p$. Then, 50th percentile is called the *mode* of X.

   (d) Let X be a continuous random variable with pdf $f(x) = \dfrac{1}{\pi(1+x^2)}, \quad -\infty < x < \infty$.
   Then, mean of X is 0. (1 + 1 + 1 + 1 marks)

4. Accidents in Delhi roads involving Blueline buses obey Poisson process with 10 per month of 30 days. In a randomly chosen month of 30 days,

   (a) What is the probability that there are exactly 6 accidents in the first 15 days?

   (b) Given that exactly 6 accidents occurred in the first 15 days, what is the probability that all the four occurred in the last 9 days out of these 15 days? (2 + 3 marks)

5. Let X be a random variable with pdf $f(x) = \frac{1}{\pi(1+x^2)}, \quad -\infty < x < \infty$. Find the CDF of the random variable

$$Y = \begin{cases} X, & |X| \geq 2 \\ 0, & |X| < 2 \end{cases}$$

(5 marks)